

線型代数学・同演習 A

4 月 21 日分 演習問題の解答

1. (a) $AB = (a_i b_{ij})_{ij}$, $BA = (a_j b_{ij})_{ij}$,

$$AB = \begin{pmatrix} a_1 b_{11} & a_1 b_{12} & a_1 b_{13} \\ a_2 b_{21} & a_2 b_{22} & a_2 b_{23} \\ a_3 b_{31} & a_3 b_{32} & a_3 b_{33} \end{pmatrix}, \quad BA = \begin{pmatrix} a_1 b_{11} & a_2 b_{12} & a_3 b_{13} \\ a_1 b_{21} & a_2 b_{22} & a_3 b_{23} \\ a_1 b_{31} & a_2 b_{32} & a_3 b_{33} \end{pmatrix}.$$

(b) $AB = \frac{n}{6}(2n^2 + 3(i - k + 1)n - 6ik + 3i - 3k + 1)_{i,k}$,

$BA = \frac{n}{2}(-2n^2 + 3(i - k - 1)n + 6ik + 3i - 3k - 1)_{i,k}$,

$$AB = \begin{pmatrix} 11 & 2 & -7 \\ 14 & 2 & -10 \\ 17 & 2 & -13 \end{pmatrix}, \quad BA = \begin{pmatrix} -11 & -2 & -17 \\ -2 & -2 & -2 \\ 7 & 10 & 13 \end{pmatrix}$$

(c) $(AB)_{ik} = \begin{cases} 0 & (i < k) \\ \sum_{k \leq j \leq i} a_{ij} b_{jk} & (i \geq k) \end{cases}, \quad (BA)_{ik} = \begin{cases} 0 & (i < k) \\ \sum_{k \leq j \leq i} b_{ij} a_{jk} & (i \geq k) \end{cases}$

$$AB = \begin{pmatrix} a_{11}b_{11} & 0 & 0 \\ a_{21}b_{11} + a_{22}b_{21} & a_{22}b_{22} & 0 \\ a_{31}b_{11} + a_{32}b_{21} + a_{33}b_{31} & a_{32}b_{22} + a_{33}b_{32} & a_{33}b_{33} \end{pmatrix},$$

$$BA = \begin{pmatrix} b_{11}a_{11} & 0 & 0 \\ b_{21}a_{11} + b_{22}a_{21} & b_{22}a_{22} & 0 \\ b_{31}a_{11} + b_{32}a_{21} + b_{33}a_{31} & b_{32}a_{22} + b_{33}a_{32} & b_{33}a_{33} \end{pmatrix}.$$

2. (1) $(x, y) = (3, 1)$, (2) $(x, y) = (2 + a, 2 - a)$, (3) $(x, y) = (a + 1, -a - 2)$.

3. $\frac{x-1}{0} = \frac{y-2}{2} = \frac{z-3}{1}$.

4. $2y + z = 3$

5. (1) $x - 2y + z = 0$, (2) $y + z = 1$, (3) $8x + 14y + 9z = 29$, (4) $bcx + acy + abz = abc$.

6. (a) $2x + y - 2z = -5$, (b) $5(x - x_0) + 2(y - y_0) - 3(z - z_0) = 0$, (c) $5x + 11y - 2z = 3$.

7. (a) $\frac{x-2}{3} = \frac{y-1}{-1} = \frac{z}{-7}$, (b) $\frac{x+2}{5} = \frac{y-\frac{7}{2}}{-4} = z$, (c) $\frac{x+4}{3} = \frac{y+2}{4} = z$.

8. (a), (b) は定義に戻って計算するだけ.

(c) ${}^tX = -X$, ${}^tY = -Y$ および ${}^t(XY) = {}^tY {}^tX$ より,

$${}^t([X, Y]) = {}^t(XY - YX) = {}^tY {}^tX - {}^tX {}^tY = YX - XY = -([X, Y]).$$