線型代数学・同演習 A

演習問題の解答

1. (1) $\begin{pmatrix} 7 & 7 & 2 \\ -3 & 8 & 1 \end{pmatrix}$, (2) $\begin{pmatrix} 1 & -3 \\ -8 & 12 \end{pmatrix}$, (3) $\begin{pmatrix} 0 & 3 \\ -1 & 1 \end{pmatrix}$, (4) $\begin{pmatrix} 46 \\ 59 \end{pmatrix}$, (5) $\begin{pmatrix} -6 \\ 10 \\ 23 \end{pmatrix}$,
(6) $\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$, (7) $\begin{pmatrix} 1 & a+b \\ 0 & 1 \end{pmatrix}$.
2. (1) $(x,y,u,v)=(2,0,3,7)$, (2) $(x,y,u,v)=(\frac{3}{2},-\frac{1}{2},4,2)$.
3. (1) $\begin{pmatrix} 2 & 2 \\ 2 & 4 \end{pmatrix}$, (2) $\begin{pmatrix} 4 & 2 \\ 2 & -4 \end{pmatrix}$, (3) $\begin{pmatrix} -4 & 4 \\ -8 & 4 \end{pmatrix}$, (4) $\begin{pmatrix} 1 & 6 \\ 0 & 4 \end{pmatrix}$.
4. $(x,a,b)=(2,3,2),\ (5,3,2)$.
5. (1) $\dfrac{1}{3}\begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$, (2) 存在しない, (3) $\dfrac{1}{ad}\begin{pmatrix} d & -b \\ 0 & a \end{pmatrix}$, (4) $\theta=\frac{2n+1}{4}\pi$ のとき存在しない,
$\theta\neq\frac{2n+1}{4}\pi$ のとき $\dfrac{1}{\cos 2\theta}\begin{pmatrix} \cos\theta & -\sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$.
6. (1) $(x,y)=(2,-1)$, (2) $(x,y)=(-1,2)$, (3) $(x,y)=(1,2)$
7. (a) $AB=(a_ib_{ij})_{ij}$, $BA=(a_jb_{ij})_{ij}$, (b) $AB=\frac{n}{2}\big(2n^2+(i-k+3)n-2ik+i-k+1\big)_{i,k}$, $BA=\frac{n}{2}\big(-2n^2+(i-k-3)n+2ik+i-k-1\big)_{i,k}$, (c) $(AB)_{ik}=0\ (i<k)$, $(AB)_{ik}=\sum_{k\leq j\leq i}a_{ij}b_{jk}\ (i\geq k)$, $(BA)_{ik}=0\ (i<k)$, $(BA)_{ik}=\sum_{k\leq j\leq i}b_{ij}a_{jk}\ (i\geq k)$.
8. (1) $(x,y)=(3,1)$, (2) $(x,y)=(-a-2,a-2)$, (3) $(x,y)=(a+1,-a-2)$.
9. $\dfrac{x-1}{0}=\dfrac{y-2}{2}=\dfrac{z-3}{3}$.
10. $2y+z=3$
11. (1) $x-2y+z=0$, (2) $y+z=1$, (3) $8x+14y+9z=29$, (4) $bcx+acy+abz=abc$.
12. (a) $2x+y-2z=-5$, (b) $5(x-x_0)+2(y-y_0)-3(z-z_0)=0$, (c) $5x+7y-2z=13$.
13. (a) $\dfrac{x-2}{3}=\dfrac{y-1}{-1}=\dfrac{z}{-7}$, (b) $\dfrac{x+2}{5}=\dfrac{y-\frac{7}{2}}{-4}=z$, (c) $\dfrac{x+4}{3}=\dfrac{y+2}{4}=z$.
14. (1) $\frac{\pi}{4}$, (2) $\frac{\pi}{3}$.
15. (1) $\begin{pmatrix} 1 & -1 \\ 2 & 2 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}$, $\begin{pmatrix} -1 & 1 \\ 2 & 2 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}$, (2) $\begin{pmatrix} 1 & -1 \\ 2 & 2 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}$, $\begin{pmatrix} -1 & 1 \\ 2 & 2 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}$, $\begin{pmatrix} -1 & -1 \\ 2 & -2 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}+$

$\begin{pmatrix}1\\2\end{pmatrix}$, $\begin{pmatrix}-1&-1\\-2&2\end{pmatrix}\begin{pmatrix}x\\y\end{pmatrix}+\begin{pmatrix}1\\2\end{pmatrix}$, $\begin{pmatrix}1&1\\2&-2\end{pmatrix}\begin{pmatrix}x\\y\end{pmatrix}+\begin{pmatrix}-1\\2\end{pmatrix}$, $\begin{pmatrix}1&1\\-2&2\end{pmatrix}\begin{pmatrix}x\\y\end{pmatrix}+\begin{pmatrix}-1\\2\end{pmatrix}$,
$\begin{pmatrix}1&-1\\-2&-2\end{pmatrix}\begin{pmatrix}x\\y\end{pmatrix}+\begin{pmatrix}0\\4\end{pmatrix}$, $\begin{pmatrix}1&-1\\-2&-2\end{pmatrix}\begin{pmatrix}x\\y\end{pmatrix}+\begin{pmatrix}0\\4\end{pmatrix}$ の計 8 個.

16. a, b を任意の実数とすると, $\begin{pmatrix}a&0\\2ab&a^2\end{pmatrix}\begin{pmatrix}x\\y\end{pmatrix}+\begin{pmatrix}b\\b^2\end{pmatrix}$, 原点は放物線 $y=x^2$ 上の任意の点に移り得る.

17. $\dfrac{x+5}{5}=\dfrac{y+2}{14}=\dfrac{z+4}{16}$.

18. (1) $\begin{pmatrix}0&-1\\-1&0\end{pmatrix}\begin{pmatrix}x\\y\end{pmatrix}+\begin{pmatrix}8\\8\end{pmatrix}$, (2) $\begin{pmatrix}\frac{1-a^2}{1+a^2}&\frac{2a}{1+a^2}\\\frac{2a}{1+a^2}&-\frac{1-a^2}{1+a^2}\end{pmatrix}\begin{pmatrix}x\\y\end{pmatrix}+\dfrac{2}{1+a^2}\begin{pmatrix}ab\\-b\end{pmatrix}$

19. (1) $\dfrac{1}{3}\begin{pmatrix}1&-2&-2\\-2&1&-2\\-2&-2&1\end{pmatrix}\begin{pmatrix}x\\y\\z\end{pmatrix}+\dfrac{16}{3}\begin{pmatrix}1\\1\\1\end{pmatrix}$, (2) $\dfrac{1}{21}\begin{pmatrix}13&16&-4\\16&-11&8\\-4&8&19\end{pmatrix}\begin{pmatrix}x\\y\\z\end{pmatrix}+\dfrac{10}{21}\begin{pmatrix}2\\-4\\1\end{pmatrix}$,

(3) $\dfrac{1}{a^2+2}\begin{pmatrix}a^2&-2&-2a\\-2&a^2&-2a\\-2a&-2a&2\end{pmatrix}\begin{pmatrix}x\\y\\z\end{pmatrix}+\dfrac{2a}{a^2+2}\begin{pmatrix}1\\1\\a\end{pmatrix}$.